**PROBLEM #4**

The solution to the one dimensional wave equation with given boundary conditions and initial conditions:

PDE:

BC:

IC:

Is given by:

Use trigonometric identities to show this solution can be expressed in the following way:

Where *R* and *S* are functions of one variable.

Firs let us begin with the sum and difference identities for sine and cosine:

If we let, and , the four equations above become:

|  |  |
| --- | --- |
| 1) |  |
| 2) |  |
| 3) |  |
| 4) |  |

Add lines 1) and 2) together to get:

Take the line above and divide both sides by two and then multiply by to get,

|  |  |
| --- | --- |
|  |  |

If we rearrange the terms to a format similar to the problem solution we get:

|  |  |
| --- | --- |
| 5) |  |

Take line 3) and subtract it from line 4) to obtain,

If we take the line above and multiply by , divide by 2, and rearrange the terms we get,

|  |  |
| --- | --- |
| 6) |  |

Add lines 5) and 6) to get,

Notice that the left hand of the equation equals the terms being summed in solution to the waved equation. If we groups terms differently we get,

Let and and , then the above line becomes:

|  |  |
| --- | --- |
| 7) |  |

Let us consider the following, if you are given we can express as a single sinusoid:

where,

If we apply this to line 7) we get,

For the first half of line 7) it can becomes,

Here we have +δ because, . Repeat this for the second half of line 7) to get,

Substitute the boxed equations into line 7) we get,

8)

Where,

Recall, and and , so line 8) becomes,

From when we added lines 5) and 6) we can conclude;

Or simply put it,

The two summations are cosine series expansions acting on and ; therefore,

Where,

And,